and for the angular motion, taking moments about the axis of the roll, θ being the angular motion corresponding to x, and so

$$x = r\theta \cdots \quad (4), \qquad \frac{d}{dt}(\rho\cdot\pi r^2\cdot k^2\dot{\theta}) = rF. \tag{5}$$

We have $k^2 = r^2/2$ (6), and from (4), $\dot{\theta} = \dot{x}/r$ (7). Substituting (6) and (7) in (5) and eliminating F from the resulting equation and (3), developing the derivatives and reducing,

$$\ddot{x} + \frac{7}{3}\cdot\frac{\dot{r}}{r}\dot{x} = \frac{2g\sin\phi}{3}. \tag{8}$$

Now, from (2), $\dot{x} = -(2\pi r/a)\dot{r}$ (9), $\ddot{x} = -(2\pi/a)(\dot{r}^2 + r\ddot{r})$ (10), and substituting these in (8) and arranging,

$$\ddot{r} + \frac{10}{3r}\dot{r}^2 = -\frac{ag\sin\varphi}{3\pi r}. \tag{11}$$

Putting $\dot{r}^2 = y$ (12), (11) is thrown into the form $(dy/dr) + Py = Q$ (13), in which $P = 20/3r$ (14), and $Q = -(2ag\sin\varphi/3\pi r)$ (15), the integral (13) being

$$y = e^{-\int Pdr}\int e^{\int Pdr}Q + Ce^{-\int Pdr}. \tag{16}$$

Substituting (14) and (15) in (16) and performing the operations indicated,

$$y = -\frac{2ag\sin\varphi}{3\pi}r^{-20/3}\cdot\frac{3}{20}r^{20/3} + Cr^{-20/3} = -\frac{2ag\sin\varphi}{3\pi} + Cr^{-20/3}. \tag{17}$$

But when $r = b$, $y = 0$, and, hence, $C = agb^{20/3}\sin\varphi/(30\pi)$, and (17) becomes

$$\dot{r}^2 = \frac{dr^2}{dt^2} = \frac{ag\sin\varphi}{30\pi}\left(\frac{b^{20/3} - r^{20/3}}{r^{20/3}}\right), \tag{18}$$

giving for the required time

$$t = \sqrt{\frac{30\pi}{ag\sin\varphi}}\int_0^b \frac{r^{10/3}dr}{\sqrt{b^{20/3} - r^{20/3}}}. \tag{19}$$

The last factor may be put into the form,

$$I = \int_0^b \frac{r^{10/3}}{b^{10/3}}\left(1 + \frac{r^{20/3}}{2b^{20/3}} + \frac{3}{8}\frac{r^{40/3}}{b^{40/3}} + \cdots\right)dr, \tag{20}$$

the series appearing to converge suitably for the degree of approximation for t in (19) by the conditions of the problem, the result though differing from that stated in the problem.

347 (Mechanics). Proposed by E. B. ESCOTT, Kansas City, Mo.

A cord $ABCD$ is suspended from points A and D which are 20 feet apart in horizontal distance. D is 4 feet lower than A. At B and C are suspended weights 100 and 200 lbs. $AB = 8$ feet, $BC = 10$ feet, $CD = 12$ feet. Find angles α, β, γ made by AB, BC, CD, respectively, with the horizontal. Also find the tensions T_1, T_2, T_3 in AB, BC, CD.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

For the equilibrium of $w_1 = 100$, resolving vertically and horizontally,

$$T_1\sin\alpha - T_2\sin\beta = w_1 = 100,$$

$$T_1\cos\alpha = T_2\cos\beta;$$

similarly for $w_2 = 200$,

$$T_3\sin\gamma + T_2\sin\beta = w_2 = 200,$$

$$T_3\cos\gamma = T_2\cos\beta.$$

The sums of the horizontal and vertical projections of AB, BC, CD give

$$8 \cos \alpha + 10 \cos \beta + 12 \cos \gamma = 20,$$

$$8 \sin \alpha + 10 \sin \beta - 12 \sin \gamma = 4.$$

These six equations furnish the theoretical solution of the problem.

The PROPOSER furnished a complete solution.

348 (Mechanics). Proposed by ALTON L. MILLER, Ann Arbor, Michigan.

If equilateral triangles be constructed on the sides of any triangle, their centers are the vertices of a new equilateral triangle. Show that the center of gravity of this new equilateral triangle coincides with the center of gravity of the original triangle.

SOLUTION BY EMMA M. GIBSON, Springfield, Mo.

Let ABC be the given triangle and let the coördinates of the vertices A, B, C referred to the rectangular axes ox and oy be (c, o), (o, b), (a, o), respectively. The equation of the line through (a, o) and (o, b) is

$$y = -\frac{b}{a}x + b. \tag{1}$$

The line from D, the third vertex of the equilateral triangle on BC, through $(a/2, b/2)$ and perpendicular to (1) is

$$y = \frac{a}{b}x + \frac{b^2 - a^2}{2b} \tag{2}$$

The line through C making an angle of 60° with (1) is

$$y = \frac{b + a\sqrt{3}}{b\sqrt{3} - a}(x - a). \tag{3}$$

Solving equations (2) and (3), the values of the coördinates of D are found to be $[(a + b\sqrt{3})/2, (b + a\sqrt{3})/2]$.

Similarly the coördinates of F and E are found to be $[(a + c)/2, \sqrt{3}(c - a)/2]$ and $[(c - b\sqrt{3})/2, (b - c\sqrt{3})/2]$, respectively.

Now the centers H, G, I of the three equilateral triangles are

$$\left(\frac{3a + b\sqrt{3}}{6}, \frac{3b + a\sqrt{3}}{6}\right), \quad \left(\frac{a + c}{2}, \frac{\sqrt{3}(c - a)}{6}\right), \quad \left(\frac{3c - b\sqrt{3}}{6}, \frac{3b - c\sqrt{3}}{6}\right),$$

respectively, since $\bar{x} = \frac{1}{3}(x_1 + x_2 + x_3)$, $\bar{y} = \frac{1}{3}(y_1 + y_2 + y_3)$. These points are the vertices of the new triangle and by the formula for the length of a line between two points, the three sides are proved equal. Hence, the new triangle is equilateral.

The coördinates of the center of gravity of the original triangle are $\bar{x} = \frac{1}{3}[a + c]$, $\bar{y} = \frac{1}{3}b$. The coördinates of the center of gravity of the new triangle are

$$\bar{x} = \frac{1}{3}\left[\frac{a + b\sqrt{3}}{2} + \frac{a + c}{2} + \frac{c - b\sqrt{3}}{2}\right] = \tfrac{1}{3}(a + c),$$

$$\bar{y} = \frac{1}{3}\left[\frac{b + a\sqrt{3}}{2} + \frac{\sqrt{3}(c - a)}{2} + \frac{b - c\sqrt{3}}{2}\right] = \tfrac{1}{3}b,$$

which are the same as those obtained for the original triangle.

Also solved by HORACE OLSON and ROGER JOHNSON.

268 (Number Theory). Proposed by FRANK IRWIN, University of California.

Show that in any arithmetical progression, whose first term a_1 and common difference d are positive integers, any required number of consecutive terms may be found, no one of which is a prime number.